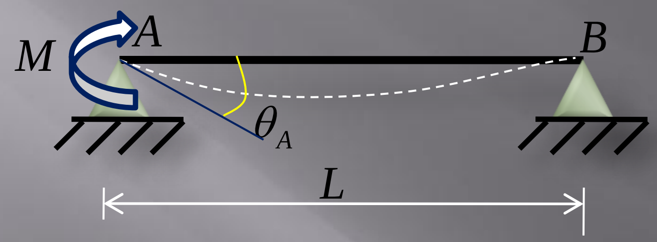
STRUCTURAL ANALYSIS

(Moment Distribution Method Contd.)

Member stiffness Factor
(for member pin supported at far end)



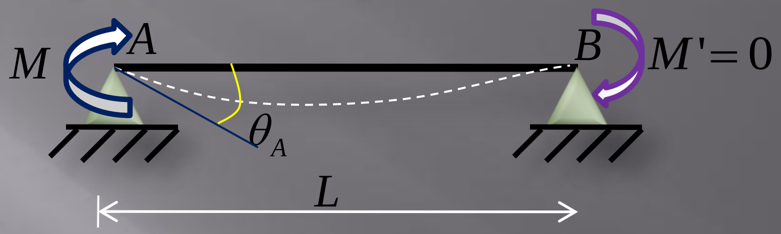
Many indeterminate beams have their far end span supported by an end pin (or roller). Application of the moment M causes the end A to rotate through an angle θ_A . The relation between moment M and θ_A is

$$M = \frac{3EI}{L}\theta_A \quad \text{end span with far end pinned/roller}$$

Stiffness factor for this beam is defined as the amount of moment M required to rotate the end A of the beam $\theta_A = 1$ rad. Thus for $\theta_A = 1$ radian, we have

$$K = \frac{3EI}{L} \quad \text{end span with far end pinned/roller}$$

Carry-Over Factor
(for member pin supported at far end)



Note that the carry-over moment is zero, since the pin at B does not support a moment. Hence the carry over factor is also zero as $M' = 0 \times M$.

Distribution Factor (Special cases)

$$DF_{AB} = \frac{K_{AB}}{\Sigma K} = \frac{K_{AB}}{\infty + K_{AB}} = 0$$

$$DF_{AB} = \frac{K_{AB}}{\Sigma K} = \frac{K_{AB}}{0 + K_{AB}} = 1$$

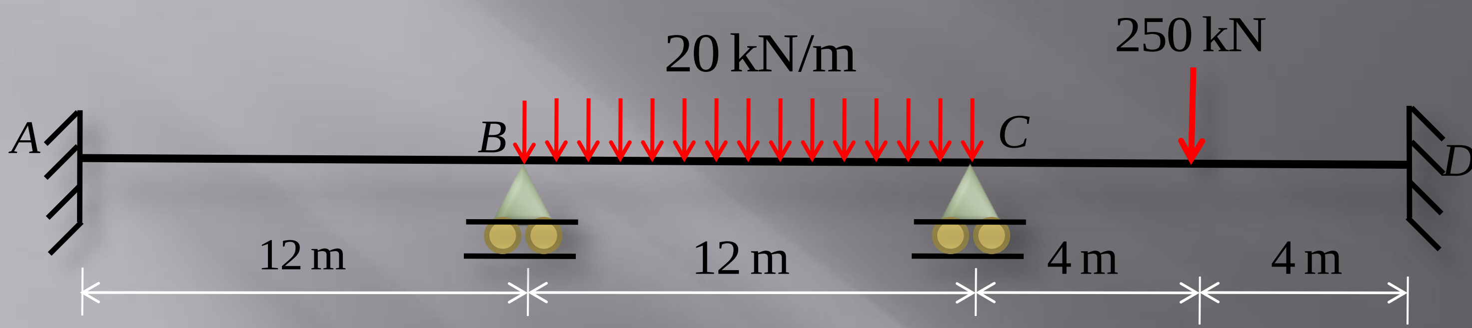
Procedure for Analysis

1. Assume all the joints are initially locked. Calculate the fixed end moments.
2. At each joint find out unbalancing moments and then determine the moment that is needed to put each joint in equilibrium (counterbalancing moment).
3. Distribute the counterbalancing moments into the connecting span at each joint according to distribution factors. This process is known as unlocking or releasing the joints.
4. Carry the moments in each span over to its other end by multiplying each moment by the carry over factor.

Note: By repeating this cycle of locking and unlocking the joints, it will be found that the moment corrections will diminish since the beam tends to achieve its final deflected shape.

Note: When small enough value for the corrections is obtained, the process of cycling should be stopped with no "carry-over" of the last moments. Each column of FEMs, distributed moments, and carry-over moments should then be added. If this is done correctly, moment equilibrium at the joints will be achieved.

Example-1



Determine the internal moments at each support of the beam shown in the Figure. *EI* is constant.

Fixed End Moments

$$(FEM)_{AB} = (FEM)_{BA} = 0$$

$$(FEM)_{BC} = -\frac{wL^2}{12} = -\frac{20\times 12^2}{12} = -240\text{ kN.m};\ (FEM)_{CB} = \frac{wL^2}{12} = \frac{20\times 12^2}{12} = 240\text{ kN.m}$$

$$(FEM)_{CD} = -\frac{PL}{8} = -\frac{250\times 8}{8} = -250\text{ kN.m};\ (FEM)_{DC} = \frac{PL}{8} = \frac{250\times 8}{8} = 250\text{ kN.m}$$

Stiffness factors for each member

$$K_{AB} = \frac{4EI}{L} = \frac{4EI}{12} = K_{BA}; \quad K_{BC} = \frac{4EI}{L} = \frac{4EI}{12} = K_{CB}$$

$$K_{CD} = \frac{4EI}{L} = \frac{4EI}{8} = K_{DC}$$

Distribution factors

$$\text{Joint A}: DF_{AB} = 0$$

$$\text{Joint B}: \ DF_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{4EI/12}{4EI/12 + 4EI/12} = 0.5$$

$$DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{4EI/12}{4EI/12 + 4EI/12} = 0.5$$

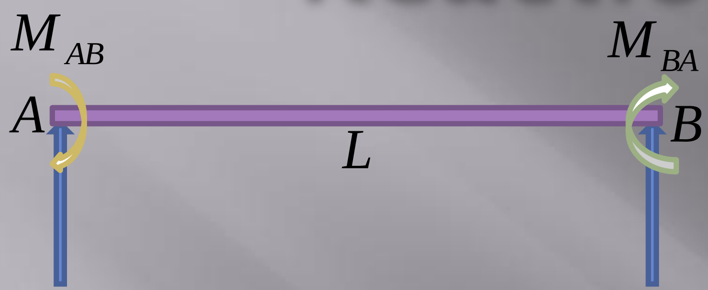
$$\text{Joint C}: \ DF_{CB} = \frac{K_{CB}}{\Sigma K} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{4EI/12}{4EI/12 + 4EI/8} = 0.4$$

$$DF_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{K_{CD}}{K_{CB} + K_{CD}} = \frac{4EI/8}{4EI/12 + 4EI/8} = 0.6$$

$$\text{Joint D}: DF_{DC} = 0$$

Joint	*A*	*B*		*C*		*D*
Member	***AB***	***BA***	***BC***	***CB***	***CD***	***DC***
DF	**0**	**0.5**	**0.5**	**0.4**	**0.6**	**0**
FEM Dist.		 120	-240 120	240 4	-250 6	250
CO Dist.	60	 -1	2 -1	60 -24	 -36	3
CO Dist.	-0.5	 6	-12 6	-0.5 0.2	 0.3	-18
CO Dist.	3	 -0.05	0.1 -0.05	3 -1.2	 -1.8	0.2
CO Dist.	-0.02	 0.3	-0.6 0.3	-0.02 0.01	 0.01	-0.9
ΣM	62.5	125.2	-125.2	281.5	-281.5	234.3

Reactions/End Shears



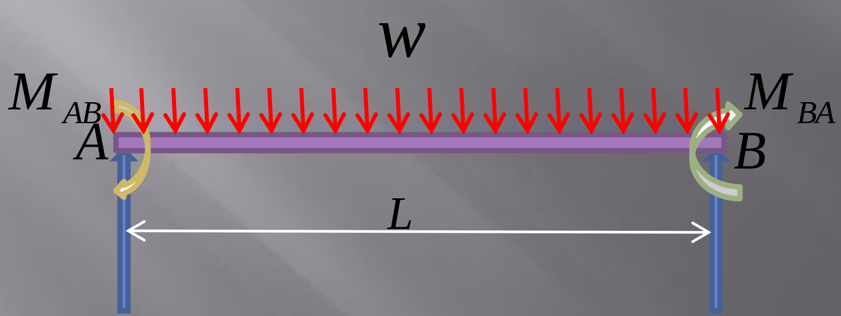
$$-\frac{(M_{AB}+M_{BA})}{L} \qquad\qquad \frac{(M_{AB}+M_{BA})}{L}$$

Reactions due to C.W. moments alone:

$$\frac{(M_{AB}+M_{BA})}{L}$$

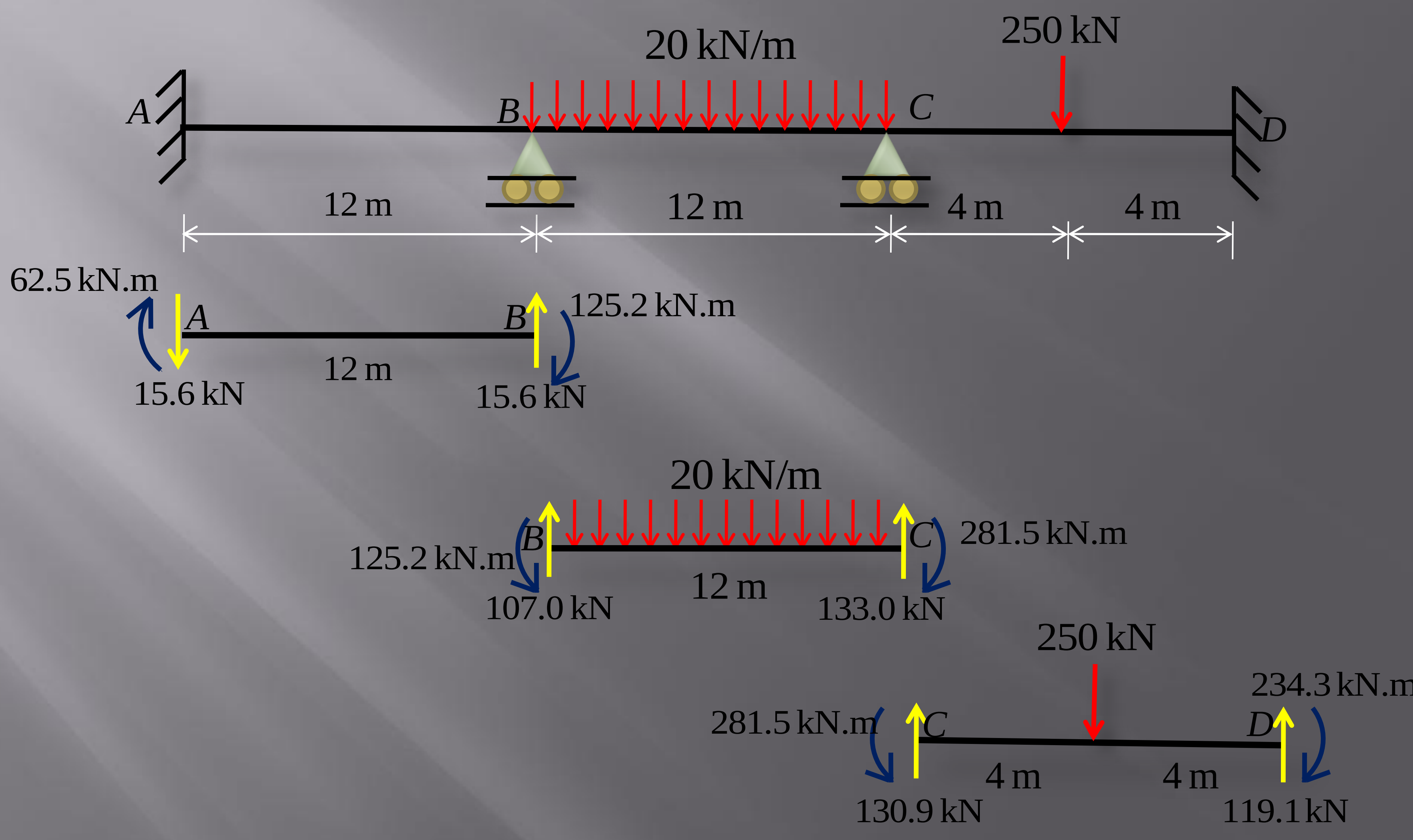
$$-\frac{(M_{AB}+M_{BA})}{L}$$

Reactions due to symmetric loads and moments:

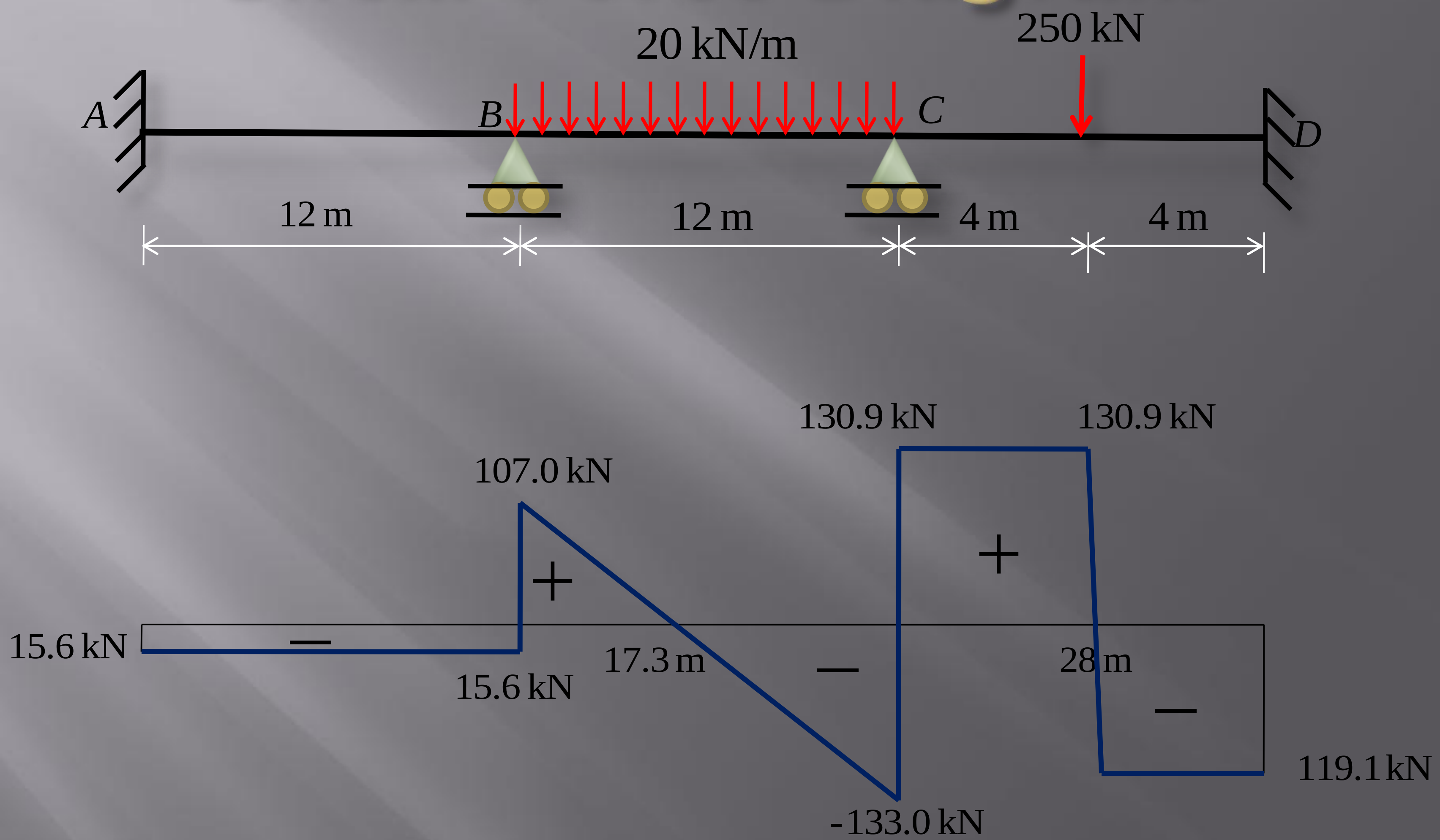


$$\left(\frac{wL}{2}\right)-\frac{(M_{AB}+M_{BA})}{L} \qquad\qquad \left(\frac{wL}{2}\right)+\frac{(M_{AB}+M_{BA})}{L}$$

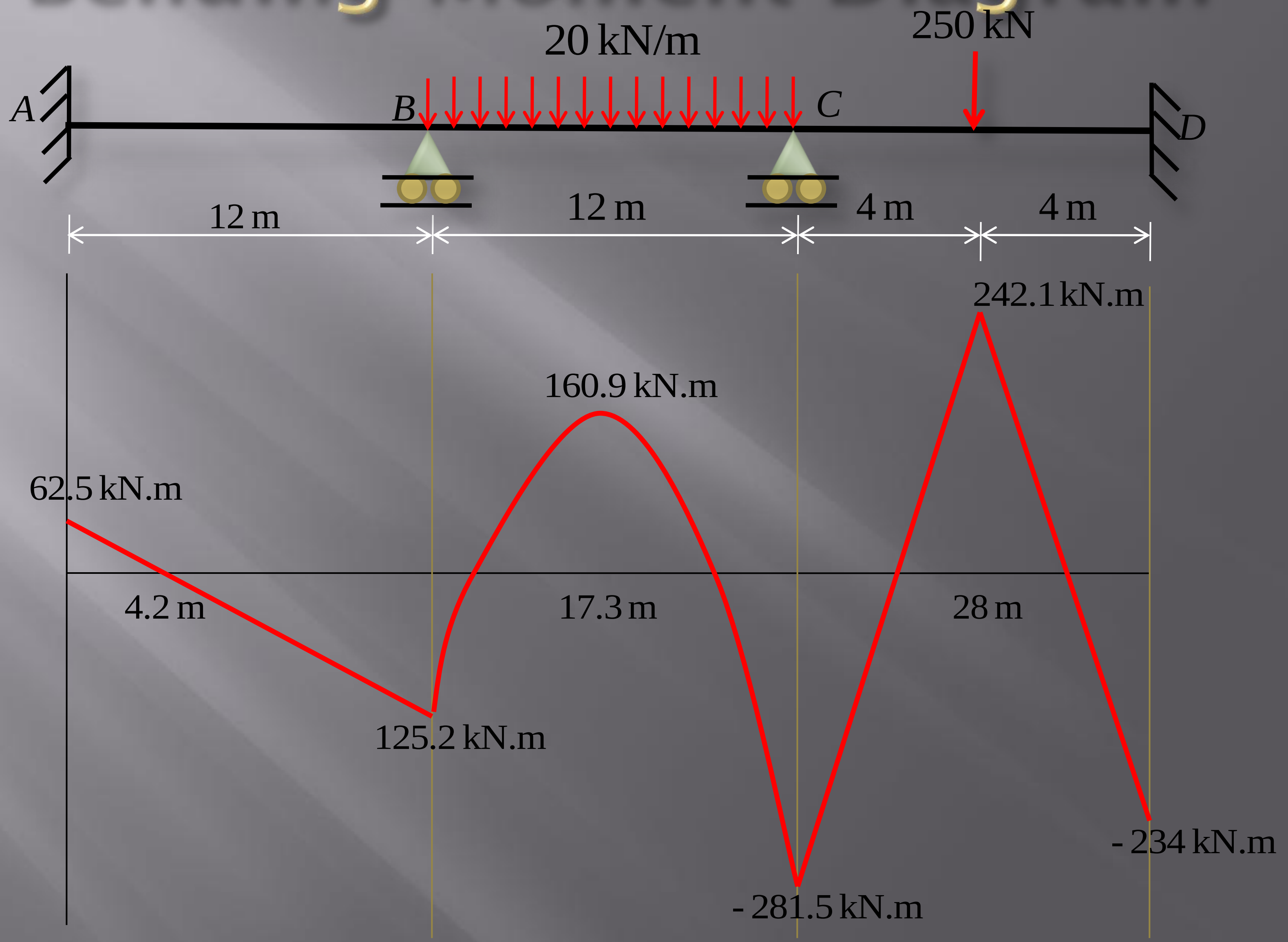
REACTIONS/END SHEARS



Shear Force Diagram



Bending Moment Diagram



STAAD OUTPUT (SFD AND BMD)